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Published in:
Physical Review Letters

DOI:
10.1103/PhysRevLett.119.130401

Publication date:
2017

Citation for published version (APA):
Beau, M., Kiukas, J., & Egusquiza, I. L. (2017). Nonexponential Quantum Decay under Environmental Decoherence. *Physical Review Letters*, *119*(13), [130401]. https://doi.org/10.1103/PhysRevLett.119.130401

Nonexponential quantum decay under environmental decoherence

M. Beau,[1,2] J. Kiukas,[3] I. L. Egusquiza,[4] and A. del Campo[1]

[1]*Department of Physics, University of Massachusetts, Boston, MA 02125, USA*
[2]*Dublin Institute for Advanced Studies, School of Theoretic al Physics, 10 Burlington Road, Dublin 4, Ireland*
[3]*Department of Mathematics, Aberystwyth University, Aberystwyth SY23 3BZ, UK*
[4]*Department of Theoretical Physics and History of Science, University of the Basque Country UPV/EHU, Apartado 644, 48080 Bilbao, Spain*

A system prepared in an unstable quantum state generally decays following an exponential law, as environmental decoherence is expected to prevent the decay products from recombining to reconstruct the initial state. Here we show the existence of deviations from exponential decay in open quantum systems under very general conditions. Our results are illustrated with the exact dynamics under quantum Brownian motion and suggest an explanation of recent experimental observations.

The exponential decay law of unstable systems is ubiquitous in Nature and has widespread applications [1–3]. Yet, in isolated quantum systems deviations occur at both short and long times of evolution [4–6]. Short time deviations underlie the quantum Zeno effect [7, 8], ubiquitously used to engineer decoherence free-subspaces and preserve quantum information. Long-time deviations are expected in any nonrelativistic systems with a ground state; they slow down the decay and generally manifest as a power-law in time [9]. Both short and long-time deviations are present as well in many-particle systems [11–16]. Indeed, the latter signal the advent of thermalization in isolated many-body systems [17, 18]. In quantum cosmology, power-law deviations constrain the likelihood of scenarios with eternal inflation [10]. They also rule the scrambling of information as measured by the decay of the form factor [19–22] in blackhole physics and strongly coupled quantum systems described by AdS/CFT, that are believed to be maximally chaotic [23].

Given a unstable quantum state $|\Psi_0\rangle$ prepared at time $t = 0$, it is customary to describe the closed-system decay dynamics via the survival probability, which is the fidelity between the initial state and its time evolution

$$\mathcal{S}(t) := |\mathcal{A}(t)|^2 = |\langle\Psi_0|\Psi(t)\rangle|^2. \tag{1}$$

Explicitly, the survival amplitude reads $\mathcal{A}(t) = \langle\Psi_0|\hat{U}(t,0)|\Psi_0\rangle$, where $\hat{U}(t,0) = T\exp(-i\int_0^t ds\hat{H}(s)/\hbar)$ is the time evolution operator generated by the Hamiltonian of the system $\hat{H}$. Short time deviations are associated with the quadratic decay

$$\mathcal{S}(t) = 1 - (t/\tau_Z)^2 + \mathcal{O}(t^3), \tag{2}$$

and are generally suppressed by the coupling to an environment that induces the appearance of a term linear in t, see, e.g. [2, 3, 24, 25]. The origin of the long-time deviations can be appreciated using the Ersak equation for the survival amplitude [4, 26, 27]

$$\mathcal{A}(t) = \mathcal{A}(t-t')\mathcal{A}(t') + m(t,t'), \tag{3}$$

that follows from the unitarity of time evolution in isolated quantum systems. The memory term reads

$$m(t,t') = \langle\Psi_0|U(t,t')\hat{Q}U(t',0)|\Psi_0\rangle, \tag{4}$$

Here, we denote the projector onto the space spanned by the initial state by $\hat{P} \equiv |\Psi_0\rangle\langle\Psi_0|$ and its orthogonal complement by $\hat{Q} \equiv 1 - \hat{P}$. As a result, the memory term $m(t,t')$ represents the formation of decay products at an intermediate time t' and their subsequent recombination to reconstruct the initial state $|\Psi_0\rangle$. The suppression of this term leads to the exponential decay law for $\mathcal{A}(t)$ and $\mathcal{S}(t)$, as an ansatz of the form $\mathcal{A}(t) = e^{-\gamma t}$ is a solution of Eq. (3) with $m(t,t') = 0$, i.e, $\mathcal{A}(t) = \mathcal{A}(t-t')\mathcal{A}(t')$. [4, 27]. In addition, using the definition of the survival probability and Eq. (3), it has been demonstrated that the long-time non-exponential behavior of $\mathcal{S}(t)$ is dominated by $|m(t,t')|^2$. The onset of long-time deviations generally occurs after many lifetimes, making their direct observation challenging. This has motivated the quest for systems where the decay is dominated by deviations and exponential decay is absent, see e.g., [28]. Nonexponential decay actually governs the dynamics in the absence of resonant states, e.g., under free dispersion.

The breakdown of unitarity can lead to exponential behavior for arbitrarily long times, as it happens in non-Hermitian systems with complex energy eigenvalues [1, 29, 30]. Nonhermitian Hamiltonians can be justified when the dynamics is restricted to a given subspace as well as in quantum measurement theory, and can delay or suppress nonexponential decay [31]. More generally, environmentally-induced decoherence is widely believed to suppress quantum state reconstruction and deviations from the exponential law [4, 32], as shown in quantum optical systems [33, 34], see as well [35]. In view of this, it came as a surprise that experimental observations consistent with nonexponential decay were reported in an open quantum system [36].

In this work, we show that nonexponential decay is ubiquitous in open quantum systems, i.e., even in the presence of environmental decoherence. We show that state reconstruction is to be expected under Markovian dynamics and is responsible for the breakdown of the exponential law. While the short-time behavior is consistent with exponential quantum decay, long-time deviations subsequently occur. These deviations are explicitly illustrated in the decay under quantum Brownian motion.

Markovian dynamics.— Consider the Hilbert space $\mathcal{H}_{SE} = \mathcal{H}_S \otimes \mathcal{H}_E$ obtained via the tensor product of the Hilbert space for the system $\mathcal{H}_S$ and that of the environment $\mathcal{H}_E$. The dynamics in $\mathcal{H}_{SE}$ is described by a unitary time evolution operator $\hat{U}_{SE}(t,0)$ generated by the full Hamiltonian, $\hat{H}_{SE} = \hat{H}_S + \hat{H}_E + \hat{H}_{int}$, where $\hat{H}_{int}$ denotes the interaction between the system and the environment. The evolution of an initially factorised state $\rho_{SE}(0) = \rho_S(0) \otimes \rho_E$ is described by the von Neumann-Liouville equation

$$\rho_{SE}(t) = \hat{U}_{SE}(t,0)\rho_S(0) \otimes \rho_E \hat{U}_{SE}(t,0)^\dagger \tag{5}$$

from which the reduced density matrix of the system $\rho_S(t) = \mathrm{Tr}_E\rho(t)$ is obtained by tracing over the environmental degrees of freedom. Under weak coupling, the evolution of the reduced dynamics of the system, $\rho_S(t) = V(t)\rho_S(0)$, is Markovian in the sense that $V(t)$ is a quantum dynamical semigroup with the composition property $V(t)V(t') = V(t+t')$ for $t, t' \geq 0$. For clarity of presentation, we shall focus on the case where $\rho_S(0)$ is pure and refer to the Supplemental material [40] for the mixed case with $\mathrm{Tr}\rho_S(0)^2 < 1$.

Short-time Markovian asymptotics.— Given a dynamical semigroup $V(t)$, the master equation associated with it is of Lindblad form [37, 38]

$$\frac{d}{dt}\rho_S = \frac{-i}{\hbar}\left[\hat{H}_s, \rho_S\right] + \sum_\alpha \gamma_\alpha \left[L_\alpha \rho_S L_\alpha^\dagger - \frac{1}{2}\left\{L_\alpha^\dagger L_\alpha, \rho_S\right\}\right], \tag{6}$$

where L_α are the Lindblad operators. Consider the fidelity between the initial pure state $\rho_S(0) = |\Psi_0\rangle\langle\Psi_0|$ and the time dependent state $\rho_S(t) = V(t)\rho_S(0)$,

$$\mathcal{S}(t) := F[\rho_S(0), \rho_S(t)] = \langle\Psi_0|\rho_S(t)|\Psi_0\rangle. \tag{7}$$

Explicit computation shows that the exact short-time asymptotics is given by [40]

$$\mathcal{S}(t) = 1 - \frac{t}{\tau_D} + O(t^2), \tag{8}$$

where

$$\tau_D = \frac{1}{\sum_\alpha \gamma_\alpha \mathrm{Cov}\left(L_\alpha, L_\alpha^\dagger\right)}, \tag{9}$$

and the covariance of two operators A and B is defined as $\mathrm{Cov}(A,B) = \langle AB\rangle - \langle A\rangle\langle B\rangle$. The requirement that the initial state is pure can be lifted, see [40]. This universal behavior of the short-time dynamics for Markovian open quantum systems is consistent with an exponential decay and suggests the identification of τ_D^{-1} with the decay rate.

By contrast, the short time asymptotic of the survival probability in $\mathcal{H}_{SE}$, defined as $\mathcal{S}_{SE}(t) := F[\rho_{SE}(0), \rho_{SE}(t)] = \left[\mathrm{Tr}\sqrt{\rho_{SE}(0)^{\frac{1}{2}}\rho_{SE}(t)\rho_{SE}(0)^{\frac{1}{2}}}\right]^2$, is characterized under Hamiltonian dynamics by a sub-exponential decay

$$\mathcal{S}_{SE}(t) = 1 - F_0 t^2/4 + O(t^3), \tag{10}$$

where the positive constant $F_0 > 0$ is the quantum Fisher information $F_0 = \mathrm{tr}[\rho_{SE}(0)\mathrm{L}_0^2]$ defined via the symmetric logarithmic derivative L_t, that satisfies $\frac{d}{dt}\rho_{SE}(t) = (\mathrm{L}_t\rho_{SE}(t) + \rho_{SE}(t)\mathrm{L}_t)/2$ [41, 42]. The subexponential decay of $\mathcal{S}_{SE}(t)$ has important applications and can be exploited, e.g. to slow down or accelerate the decay [43, 44]. While it is known that the Markovian master equation fails generally at very short-times, within the realm of its validity short-time deviations are absent. In what follows, we shall focus on the nonexponential behavior in the subsequent dynamics, under Eq. (6).

Quantum state reconstruction under quantum dynamical semigroups.— Using the composition property of dynamical semigroups it is possible to derive an analogue of the Ersak equation (3) for open quantum systems. This generalization requires a formulation in terms of probabilities, simplifying the interpretation of the analogue of the memory term in the unitary case, (4). Indeed, explicit computation yields

$$\begin{aligned}\mathcal{S}(t) &= \mathrm{Tr}[\hat{P}V(t)\rho_S(0)] &(11)\\ &= \mathrm{Tr}[\hat{P}V(t-t')(\hat{P}+\hat{Q})V(t')\rho_S(0)(\hat{P}+\hat{Q})] &(12)\\ &= \mathcal{S}(t-t')\mathcal{S}(t') + M(t,t'), &(13)\end{aligned}$$

where we have used the fact that $\hat{P}\rho_S(t')\hat{P} = S(t')\hat{P}$ and introduced the memory term

$$\begin{aligned}M(t,t') = &\mathrm{Tr}\left\{\hat{P}V(t-t')\left[\hat{Q}\left(V(t')\rho_S(0)\right)\hat{Q}\right]\right\}\\ &+\mathrm{Tr}\left\{\hat{P}V(t-t')\left[\hat{Q}\left(V(t')\rho_S(0)\right)\hat{P}\right]\right\}\\ &+\mathrm{Tr}\left\{\hat{P}V(t-t')\left[\hat{P}\left(V(t')\rho_S(0)\right)\hat{Q}\right]\right\}. \qquad (14)\end{aligned}$$

Equation (13) is the generalization of the Ersak equation [26] for quantum Markovian dynamics for an initial pure state $\rho_S(0) = \hat{P}$; see [40] for the mixed case. The first term in the rhs of the memory term $M(t,t')$ (14) represents the conditional probability to find the time-evolving state at time t in the space spanned by the initial state, provided it was found in the orthogonal subspace at an intermediate time t', i.e., that it had fully decayed. The remaining two crossed terms result from interference involving state reconstruction, i.e., the coherences of the density matrix for this P-Q decomposition. When the memory term vanishes identically, $M(t,t') = 0$, the generalized Ersak equation (13) dictates exponential decay, $\mathcal{S}(t) = e^{-\gamma t}$, that satisfies $\mathcal{S}(t) = \mathcal{S}(t-t')\mathcal{S}(t')$. As in the unitary case [4], any deviation from an exponential decay law for $\mathcal{S}(t)$ arises due to the state reconstruction of the initial state $\rho_S(0)$ from the decay products found at the intermediate time t', during the evolution between t' and t. The memory term does not generally vanish, justifying the ubiquity of deviations from the exponential decay law in open quantum systems.

Universality of long-time subexponential decay in Markovian quantum systems.— We next establish that the long-time decay of the survival probability in open quantum systems, in particular also Markovian, is generally not exponential. We focus on Hamiltonians $\hat{H}_{SE}$ with a continuous energy spectrum $E \in [E_0, \infty)$. In general, each energy eigenvalue may have several (improper) eigenstates associated with it, but this multiplicity does not play any role in the following, and hence we assume for simplicity that there is just one eigenstate $|E\rangle$ for each E. One can easily check that the same argument works also in the general case.

An initial state of the composite system of factorized form $\rho_{SE}(0) = \rho_S(0)\otimes\rho_E$ will have coherences in the energy representation. We write it in its diagonal representation $\rho_{SE}(0) = \sum_j \lambda_j|\lambda_j\rangle\langle\lambda_j|$ where the occupation numbers $\lambda_j \geq 0$ and $|\lambda_j\rangle \in \mathcal{H}_{SE}$. We next exploit a purification of $\rho_{SE}(0)$ in an enlarged Hilbert space $\mathcal{H}_{SE}\otimes\mathcal{H}_R$; we take $\mathcal{H}_R = \mathcal{H}_{SE}$ and define

$$|\Psi_{SER}(0)\rangle = \sum_j \sqrt{\lambda_j}|\lambda_j\rangle\otimes|\lambda_j\rangle, \tag{15}$$

where the finiteness of the sum $\sum_j \lambda_j = \mathrm{tr}[\rho_{SE}(0)] = 1 < \infty$ ensures that this belongs to the Hilbert space. Denoting by $|E\rangle \in \mathcal{H}_{SE}$ the energy eigenkets of $\hat{H}_{SE}$, we consider the time evolution operator

$$\hat{U}_{SE}(t,0)\otimes 1_R = \int_{E_0}^{\infty} dE e^{-iEt/\hbar}|E\rangle\langle E|\otimes 1_R. \tag{16}$$

The dynamics of the purified state is then described by

$$\begin{aligned}|\Psi_{SER}(t)\rangle &= (\hat{U}_{SE}(t,0)\otimes 1_R)|\Psi_{SER}(0)\rangle\\ &= \sum_j \int_{E_0}^{\infty} dE\,\sqrt{\lambda_j}\langle E|\lambda_j\rangle e^{-iEt/\hbar}|E\rangle\otimes|\lambda_j\rangle, \end{aligned}\tag{17}$$

and we stress that $\Psi_{SER}(t)$ is indeed a purification of the physical system-environment state $\rho_{SE}(t) = \hat{U}_{SE}(t,0)\rho(0)\hat{U}_{SE}(t,0)^\dagger$ (and hence also of the system state $\rho_S(t)$) at each time t. In terms of $\Psi_{SER}(t)$ we introduce the survival amplitude between the initial purified state at $t=0$ and that at any time $t\geq 0$, i.e.,

$$\begin{aligned}\mathcal{A}_{SER}(t) &:= \langle\Psi_{SER}(0)|\Psi_{SER}(t)\rangle\\ &= \sum_j \int_{E_0}^{\infty} dE\lambda_j|\langle E|\lambda_j\rangle|^2 e^{-iEt/\hbar}\\ &= \int_{-\infty}^{\infty} dE\,\varrho_{SE}(E)\,e^{-iEt/\hbar}, \end{aligned}\tag{18}$$

where we have denoted the energy distribution of the initial state by $\varrho_{SE}(E) = |\langle\Psi_{SE}(0)|E\rangle|^2 = \sum_j \lambda_j|\langle E|\lambda_j\rangle|^2$, a function that vanishes for any $E < E_0$. We note that the combination of the sum and the integral converges absolutely, hence the order may be interchanged. The semi-finiteness of $\varrho_{SE}(E)$ dictates the analytic properties of its Fourier transform $\mathcal{A}_{SER}(t)$. In particular, the Paley-Wiener theorem [9, 45] imposes the convergence of the integral

$$\int_{\mathbb{R}} dt \frac{|\log|\mathcal{A}_{SER}(t)||}{1+(t/t_0)^2} < \infty\,, \tag{19}$$

where t_0 is any constant with dimensions of time. As a result, the survival probability in the enlarged Hilbert space decays slower than any exponential function $e^{-\alpha t}$ for large t,

$$|\mathcal{A}_{SER}(t)|^2 \geq Ce^{-\gamma t^q},\ \text{ with } C,\gamma>0, \text{ and } q<, \tag{20}$$

In order to connect this with the survival probability $\mathcal{S}(t)$ we use Uhlmann's theorem, which states that the fidelity $\mathcal{S}_{SE}(t) := F[\rho_{SE}(0),\rho_{SE}(t)]$ equals the maximal fidelity between all possible purifications of $\rho_{SE}(0)$ and $\rho_{SE}(t)$ [46, 47]. Since $\Psi_{SER}(t)$ is a purification of $\rho_{SE}(t)$, we get $\mathcal{S}_{SE} \geq |\mathcal{A}_{SER}(t)|^2$ for each $t\geq 0$. Finally, using the monotonicity (or non-contractivity) of the fidelity, $\mathcal{S}(t)\geq\mathcal{S}_{SE}$, it follows that the the survival probability of the system $\mathcal{S}(t)$ cannot decay faster than the fidelity in $\mathcal{H}_{SE}$. We note that the theorem [47] is not restricted to a finite-dimensional setting, and hence works in our case. From (20) we then get

$$\mathcal{S}(t) \geq Ce^{-\gamma t^q},\ \text{ with } C,\gamma>0, \text{ and } q<1. \tag{21}$$

which is the main result of this section and generalizes the corresponding result for closed systems [4]. To summarize, the existence of the ground-state E_0 in the composite system $\hat{H}_{SE}$ makes $\varrho_{SE}(E)$ semi-finite and dictates the long-time behavior of the survival probability $\mathcal{S}(t)$ of the system $\rho_S(0)$ under Markovian evolution. While the result holds in full generality, it is intended for the case of an infinite-dimensional system Hilbert space $\mathcal{H}_S$. In fact, in the finite-dimensional case $\mathcal{S}(t)$ is not even expected to vanish at long times, since $\rho_S(t)$ typically tends to a full-rank stationary state. Only quantities such as coherences can then exhibit exponential decay (as in [35]).

We have established that deviations from exponential decay are to be expected under environmental decoherence at both short and long-times of evolution. In particular, a power-law behavior as experimentally reported in [36] is consistent with open quantum dynamics. Yet, a nearly exponential decay is not excluded by the Paley-Wiener theorem [48]. We next focus on a paradigmatic example of open quantum dynamics on an infinite-dimensional space and that is governed by nonexponential behavior: quantum Brownian motion.

Nonexponential decay under quantum Brownian motion.— Consider a single quantum particle of mass m in contact with a thermal bath. We assume weak coupling between the particle and the bath, a large temperature regime, and the Born-Markov approximation. The dynamics is then well described by the Caldeira-Leggett model [38, 39]

$$\frac{d}{dt}\rho_S(t) = \frac{-i}{\hbar}\left[H,\rho_S(t)\right] - \frac{i\gamma}{\hbar}[x,\{p,\rho_S(t)\}] - D\left[x,\left[x,\rho_S(t)\right]\right]\,, \tag{22}$$

with $H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$, and where the coupling constant $D = 2m\gamma k_B T/\hbar^2$ depends explicitly on the temperature T of the bath and on the damping constant γ. Eq. (9) for the decoherence time leads to

$$\tau_D = \frac{\lambda_\beta^2}{2\gamma\Delta x^2}\,, \tag{23}$$

when $\tau_D \ll \gamma^{-1}$, in terms of the de Broglie thermal wavelengths $\lambda_\beta^2 = \hbar^2/(2mk_BT)$ and Δx^2 the variance of the initial pure state. Consistently with the high temperature regime, we assume that the characteristic time scale of the system $\tau_c \equiv m\Delta x^2/\hbar$ is large compared to the thermal bath characteristic time $\tau_\beta \equiv \hbar/(k_BT)$. Equivalently, $\lambda_\beta \ll \Delta x$ which entails $\tau_D \ll \tau_R = \gamma^{-1}$.

For the sake of illustration, consider the initial pure Gaussian state

$$\rho_S(x, y; 0) = \sqrt{\frac{1}{\pi\sigma^2}} \exp\left[-\left(\frac{x^2+y^2}{2\sigma^2}\right)\right], \tag{24}$$

with $\Delta x = \sigma/\sqrt{2}$. We find that there are three distinct regimes in the decay dynamics:

(i) At short times $t \ll \tau_D \ll \tau_R$, the survival probability behaves as $S(t) \approx 1 - t/\tau_D$.

(ii) Subsequently, an intermediate regime sets in for $\tau_D \ll t \ll \tau_R$ when the system is undamped and experiences decoherence. The off-diagonal density matrix decays exponentially in time [49] $\rho_S(x, y; t) \approx \rho_S(0, 0; t) \times \exp\left[-Dt(x-y)^2\right] \exp\left[i\phi(x, y)\right]$, where ϕ is the complex part of the phase. In this regime, the density profile (this is, the diagonal part of $\rho_S(x, y; t)$) has the asymptotics $\rho_S(x, x; t) \approx \frac{1}{\sqrt{2\pi\Delta x(t)^2}} \exp\left(-\frac{x^2}{2\Delta x(t)^2}\right)$, where the normalization factor of the density matrix scales as [38] $\Delta x(t) \approx \sqrt{\frac{4Dt^3}{3}}$. Hence, we find from the long-time asymptotics of density matrix $\rho_S(x, y; t) \approx \sqrt{\frac{2}{Dt}}\delta(x-y)\rho_S(x, x; t)$ that the survival probability (7) simplifies to $\mathcal{S}(t) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \rho_S(x, y; 0)\rho_S(y, x; t) \approx \sqrt{\frac{2}{Dt}} \int_{-\infty}^{\infty} dx \rho_S(x, x; 0)\rho_S(x, x; t) \approx \sqrt{\frac{2}{Dt}} \frac{1}{\Delta x(t)}$ as $\Delta x(t)^2 \gg \Delta x$. This leads to the power-law asymptotic behavior

$$\mathcal{S}(t) \approx \sqrt{3}\frac{m}{D\hbar t^2} = \sqrt{3}\frac{\tau_\beta \tau_R}{t^2}, \tag{25}$$

which is independent of Δx and the initial width of the Gaussian wave packet. The two relevant time scales are set by $\tau_\beta \equiv \hbar/(k_B T)$ and $\tau_R = \gamma^{-1}$.

(iii) The final and third stage of the dynamics occurs when the evolution time becomes large compared to the relaxation time of the environment $t \gg \tau_R \gg \tau_D$. The system becomes overdamped and the off-diagonal density matrix converges to a stationary solution $\rho_S(x, y; t) \approx \rho_S(0, 0; t) \times \exp\left(-\frac{(x-y)^2}{2\lambda_\beta^2}\right) \exp\left[i\phi(x, y)\right]$, where the normalization factor is given by the diffusion variance $\Delta x(t) \approx \sqrt{\frac{D\hbar^2}{m\gamma^2} t} = \sqrt{\frac{2k_B T}{m\gamma} t}$, see [38]. Using similar arguments to those for (ii) we find the long-time power-law scaling

$$\mathcal{S}(t) \approx \frac{\lambda_\beta}{\Delta x(t)} \approx \frac{\hbar\gamma}{2k_B T}\frac{1}{\sqrt{\gamma t}} = \frac{\tau_\beta}{\tau_R}\sqrt{\frac{\tau_R}{t}}. \tag{26}$$

Equations (25)-(26) can also be derived from an asymptotics analysis of the exact $\mathcal{S}(t)$, found via the Feynman-Vernon influence functional [38, 39, 50] and the multi-dimensional Gaussian integral method; see [40]. As anticipated, the asymptotic behavior predicted by equations (25)-(26) does not depend on the initial spreading Δx. A direct application of our findings is the estimation of the damping coefficient γ and the temperature T of the system, that can be extracted from the survival probability (7) upon identification of the two time scales $\tau_\beta = \hbar/(k_B T)$ and $\tau_R = \gamma^{-1}$. To do this it suffices to find the intercepts of the two asymptotic lines obtained in a log-log plot, see Fig 1 and equations (25)-(26).

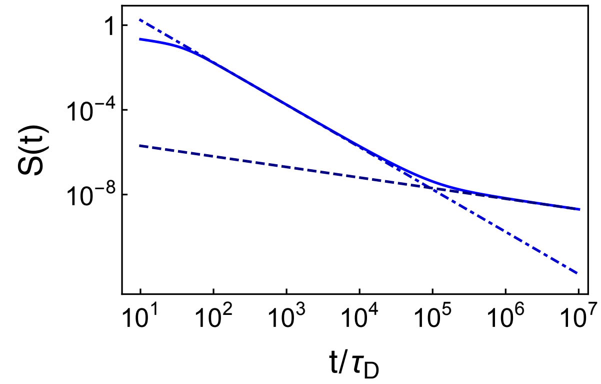

FIG. 1. **Decay of the survival probability under quantum Brownian motion.** The change in the power-law governing the survival probability of a pure initial Gaussian state (solid line) is shown in a log-log scale as a function of time in units of the decoherence time. The long-time asymptotic expressions for $t \gg \tau_D$ in the two distinct regimes $\gamma t \ll 1$ (dotted-dashed line) and $\gamma t \gg 1$ (dashed line) are also shown, with $\gamma = 10^{-3}$ and $D\sigma^2 = 100$, with $\hbar/m\sigma^2$ as a unit of frequency.

We next consider another prominent example -the quantum decay of a Schrödinger cat state- that supports the idea of universality of the long-time asymptotic behavior (25)-(26). Consider a pure state $\rho_S(x, y; 0) = \psi_0(x)\psi_0(y)^*$ made of a superposition of two Gaussian wave packets centered respectively at $x = -r$ and $x = +r$, i.e.,

$$\psi_0(x) = \mathcal{N}_\sigma\left[e^{-\frac{(x-r)^2}{2\sigma^2}} + e^{-\frac{(x+r)^2}{2\sigma^2}}\right], \tag{27}$$

where the normalization factor is $\mathcal{N}_\sigma = \left[2\sqrt{\pi\sigma^2}(1 + e^{-\frac{r^2}{\sigma^2}})\right]^{-1/2}$. The arguments we used for the Gaussian state (24) apply as well to the cat state (27). Inserting the expression for the variance of the position $\Delta x^2 = \sigma^2/2 + r^2/\left(1 + e^{-\frac{r^2}{\sigma^2}}\right)$ into equation (9), we reproduce the prediction by Zurek, $\tau_D = \lambda_\beta^2/(2\gamma r^2)$, in the limit $r \gg \sigma$ [49]. In the limit $r \ll \sigma$, we find $\tau_D = \lambda_\beta^2/(\gamma\sigma^2) = 1/(D\sigma^2)$ which agrees with the Bedingham-Halliwell decoherence time derived from the short-time asymptotics in [24]. The intermediate cases $r \sim \sigma$ offer new regimes for any initial state with a finite variance Δx. While the decoherence time defines the long-time scaling, it does not appear in the expression of the asymptotic survival probability (25)-(26). This remarkable result combined with our previous findings for Gaussian states of the form (24) supports the idea of universality of long-time asymptotics, that should be experimentally testable. The change in the power-law scaling of $\mathcal{S}(t) \propto 1/t^2$ to $\mathcal{S}(t) \propto 1/\sqrt{t}$ is demonstrated in in Fig 1 for for $r = 0$. We obtain similar plots for $r > 0$, that indicate the universal behavior of the long-time quantum decay of the survival probability.

Summary.— We have shown that the decay dynamics of open quantum systems generally exhibits deviations from exponential decay in the presence of environmental decoherence. These deviations result from the reconstruction of the initial state from the decay products formed during the course of the evolution. While the short-time quantum decay under Markovian dynamics is consistent with an exponential law, the long-time evolution is characterized by a sub-exponential decay whenever a ground state exists for the system-environment complex, e.g., in a nonrelativistic setting. We have demonstrated the existence of these deviations in quantum Brownian motion, a setting amenable to experimental investigations. Our study is expected to find broad applications across a wide variety of fields. Prominent instances include the analysis of decoherence dynamics, collapse models in quantum measurement theory, thermalization and information scrambling in open quantum systems, and quantum cosmology.

Acknowledgments.— Funding support from UMass Boston (project P20150000029279), ESF (POLATOM-5052) and the John Templeton Foundation is acknowledged. I.L.E. acknowledges funding from Spanish MINECO/FEDER Grant No. FIS2015-69983-P, and Basque Government IT986-16. AdC acknowledges the hospitality of the Centre for Quantum Technologies at the National University of Singapore during the completion of this work.

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